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T. B. C. : CS – 11/20

**Test Booklet Series**

**B**

Serial No. 238362

# TEST BOOKLET

## O. C. S. Preliminary Examination

## (MATHEMATICS)

*Time Allowed : 2 Hours*

*Maximum Marks : 300*

### : INSTRUCTIONS TO CANDIDATES :

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET **DOES NOT** HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET OF SAME SERIES ISSUED TO YOU.
2. ENCODE YOUR OPTIONAL SUBJECT CODE AS MENTIONED ON THE BODY OF YOUR ADMISSION CERTIFICATE AND ADVERTISEMENT AT APPROPRIATE PLACES ON THE ANSWER SHEET.
3. ENCODE CLEARLY THE TEST BOOKLET SERIES **A, B, C** OR **D**, AS THE CASE MAY BE, IN THE APPROPRIATE PLACE IN THE ANSWER SHEET USING BALL POINT PEN (BLUE OR BLACK).
4. You have to enter your **Roll No.** on the Test Booklet in the Box provided alongside. **DO NOT** write *anything else* on the Test Booklet.
5. This Test Booklet contains **100** items (questions). Each item (question) comprises four responses (answers). You will select the correct response (answer) which you want to mark (darken) on the Answer Sheet. In case, you feel that there is more than one correct response (answer), mark (darken) the response (answer) which you consider the best. In any case, choose **ONLY ONE** response (answer) for each item (question).
6. You have to mark (darken) all your responses (answers) **ONLY** on the **separate Answer Sheet** provided, by **using BALL POINT PEN (BLUE OR BLACK)**. See instructions in the Answer Sheet.
7. All items (questions) carry equal marks. All items (questions) are compulsory. Your total marks will depend only on the number of correct responses (answers) marked by you in the Answer Sheet. There will be negative markings for wrong answers. **25 percent of marks allotted to a particular item (question) will be deducted as negative marking for every wrong response (answer).**
8. Before you proceed to mark (darken) in the Answer Sheet the responses to various items (questions) in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per the instructions in your **Admission Certificate**.
9. After you have completed filling in all your responses on the Answer Sheet and after conclusion of the examination, you should hand over to the Invigilator the *Answer Sheet* and the *Test Booklet* issued to you. Yor are allowed to take with you the candidate's copy/second page of the Answer Sheet, after completion of the examination, for your reference.

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SEAL

1. Let S be the set of all lines in 3 dimensional space. A relation $\rho$ is defined on S by "$\ell \rho m$ if and only if $\ell$ lies on the plane of m" for $\ell, m \in S$. Then $\rho$ is :

(a) Not reflexive but symmetric and transitive

(b) Reflexive and transitive but not symmetric

(c) Reflexive and symmetric but not transitive

(d) Equivalence relation

2. How many reflexive relations possible on a set of 3 elements ?

(a) $2^6$

(b) $2^3$

(c) 16

(d) 0

3. $f : R^+ \rightarrow R$ and $g : R^+ \rightarrow R$ are given by $f(x) = 1 + \frac{x}{|x|}$ and $g(x) = 2$ respectively. Then :

(a) $f + g = 1$

(b) $f - g = 1$

(c) $F^* g = 1$

(d) $f/g = 1$

4. Let $A = \{x \in R : -1 \leq x \leq 1\}$ and $f : A \rightarrow A$ be a function defined by $f(x) = x\,|x|$. Then f is :

(a) Injective but not surjective

(b) Surjective but not injective

(c) Neither injective nor surjective

(d) Bijective

5. Consider S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Let $x \leq y$ means that x is a divisor of y. Then the maximal element or elements in the poset $(S, \leq)$ is / are :

(a) 10

(b) 6, 7, 8, 9, 10

(c) 7, 8, 9, 10

(d) 8, 9, 10

6. Let $\rho$ be a relation on a non-empty set A. Then which of the following is true ?

(a) If $\rho$ is symmetric and transitive then $\rho$ is reflexive

(b) If $\rho$ is symmetric and reflexive then $\rho$ is transitive

(c) If $\rho$ is reflexive and transitive then $\rho$ is symmetric

(d) None of the above

7. Let $g : R \to Z$ is defined as $g(x) = \frac{x}{2}$

and $f : R \to Z$ is defined as $f(x) = \left[\frac{x^2}{2}\right]$

where [u] denotes the greatest integer $\leq u$, then gof (x) is equal to :

(a) $\left[\frac{x^2}{4}\right]$

(b) $\left[\frac{\sqrt{x}}{4}\right]$

(c) $\left[\frac{x^2}{8}\right]$

(d) None of the above

8. If $f : Z \to Z$ be a function defined by $f(x) = ax + b$ for certain integers a and b, then fof is an identity function if and only if :

(a) $a = \pm 1, b = 0$

(b) $b = 0$

(c) $a = 1, b = -1$

(d) $a = 1$

9. If a, b, c, d are rational and x is irrational then $\frac{ax+b}{cx+d}$ is :

(a) Always rational

(b) Always irrational

(c) May be rational or irrational

(d) None of the above

10. The supremum and infimum of $S = \{x : 3x^2 - 10x + 3 > 0\}$ are respectively :

(a) $3, \frac{1}{3}$

(b) $\infty, -\infty$

(c) $\frac{1}{3}, -\infty$

(d) $\infty, 3$

11. If A and B be the two non empty sets bounded above and $C = \{x + y : x \in A, y \in B\}$, then :

(a) Sup C = Sup A + Inf B

(b) Sup C = Inf A + Sup B

(c) Sup C = Inf A + Inf B

(d) Sup C = Sup A + Sup B

12. Let $x_{n+1} = x_n(2 - x_n)$, $x_n > 0 \ \forall n$, $0 < x_1 < 1$, then :

(a) $\{x_n\}$ is monotonically increasing sequence and bounded

(b) $\{x_n\}$ is monotonically increasing sequence and unbounded

(c) $\{x_n\}$ is monotonically decreasing sequence and bounded

(d) $\{x_n\}$ is monotonically decreasing sequence and unbounded

13. If $\left(\frac{1-i}{1+i}\right)^{100} = a + ib$, then :

(a) $a = 2, b = -1$

(b) $a = 1, b = 0$

(c) $a = 0, b = 1$

(d) $a = -1, b = 2$

14. $\left\{\sqrt{7}, \sqrt{7\sqrt{7}}, \sqrt{7\sqrt{7\sqrt{7}}}, ..........\right\}$ converges to :

(a) 7

(b) $\sqrt{7}$

(c) $\sqrt{7\sqrt{7}}$

(d) $\sqrt{7\sqrt{7\sqrt{7}}}$

15. The equation $z\bar{z} - iz + i\bar{z} - 3 = 0$ describes :

(a) A straight line

(b) An ellipse

(c) A circle

(d) A pair of straight line

16. Which of the following is true ?

(a) $\overline{(z_1 + z_2)} = \overline{z_1} + \overline{z_2}$

(b) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$

(c) $\overline{(z_1 z_2)} = \overline{z_1}\,\overline{z_2}$

(d) All of the above

17. By which condition $\ell x + my = 1$ should touch $(ax)^n + (by)^n = 1$.

(a) $\left(\frac{\ell}{a}\right)^{\frac{n}{n-1}} + \left(\frac{m}{b}\right)^{\frac{n}{n-1}} = 1$

(b) $\left(\frac{\ell}{a}\right)^{\frac{n-1}{n}} + \left(\frac{m}{b}\right)^{\frac{n-1}{n}} = 1$

(c) $\left(\frac{\ell}{b}\right)^{\frac{n-1}{n}} + \left(\frac{m}{a}\right)^{\frac{n-1}{n}} = 1$

(d) $\left(\frac{\ell}{b}\right)^{\frac{n}{n-1}} + \left(\frac{m}{a}\right)^{\frac{n}{n-1}} = 1$

18. If $f(x) = e^x$ and $g(x) = \log_e x$, then $(gof)'(x)$ is equal to :

(a) 0

(b) e

(c) 1

(d) $e^{-1}$

19. If A and B are two ideals of a ring $(R, +, \cdot)$ such that $A \cap B = \{0\}$, then AB is :

(a) $\phi$

(b) A singleton set

(c) A pair set

(d) R

20. If f is a ring homomorphism from the ring of integers to the ring of even integers, then f(2006) is :

(a) 0

(b) 2

(c) 2006

(d) None of the above

21. For the Klein's four group $(V_4, 0)$ consider the following two assertions :

(I) $V_4$ is a vector space over the field of residue-classes modulo 2.

(II) $V_4$ is a vector space over the field of residue-classes modulo 3.

Then :

(a) Both I and II are correct

(b) Both I and II are incorrect

(c) I is correct but II is incorrect

(d) II is correct but I is incorrect

22. In $\mathbb{R}^3(\mathbb{R})$ the linear span of the set $\{(1, 1, -2), (0, -1, 1), (3, -2, -1)\}$ is :

(a) $\mathbb{R}^3$

(b) A plane

(c) A line

(d) None of the above

23. If $A = \{(x, y, z) : x, y, z \in \mathbb{R} \text{ and } y + 2z = 0\}$ and $B = \{(x, y, z) : x, y, z \in \mathbb{R} \text{ and } x + y + z = 0\}$ are subspaces of the vector space $\mathbb{R}^3(\mathbb{R})$, then dimension of the linear sum of A and B is :

(a) 3

(b) 2

(c) 1

(d) 0

24. If $W_1$ and $W_2$ are the two subspaces of a vector space V over a field F such that $\dim V = 20$, $\dim W_1 = 19$, $\dim W_2 = 17$ and $W_2$ is not a subset of $W_1$, then the dimension of $W_1 \cap W_2$ is :

(a) 17

(b) 14

(c) 15

(d) 16

25. If $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 1 & 5 \end{pmatrix}$ and $q = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 5 & 4 & 1 \end{pmatrix}$ are two permutations on five symbols 1, 2, 3, 4, 5, then :

(a) Both p and q are odd

(b) Both p and q are even

(c) p is odd but q is even

(d) p is even but q is odd

26. The number of elements which are their own inverses in a group having 15 elements is :

(a) 5

(b) 3

(c) 2

(d) 1

27. The number of non-trivial proper normal subgroups of Hamiltonian group is :

(a) 4

(b) 3

(c) 2

(d) 1

28. If f be a group homomorphism from the additive group of rational numbers to the additive group of integers, then $f\left(\frac{2}{3}\right)$ is equal to :

(a) 1

(b) 0

(c) 2

(d) None of the above

29. In the set of real numbers $\mathbb{R}$ two internal binary operations $\oplus$ and $\odot$ are defined by $a \oplus b = a + b + 1$ and $a \odot b = ab + a + b$. What are the zero element and unit element of the ring $(\mathbb{R}, \oplus, \odot)$ respectively ?

(a) 0, 1

(b) 1, 0

(c) 0, – 1

(d) – 1, 0

30. In the set $\mathbb{Z} \times 2\mathbb{Z}$ two internal binary operations $\oplus$ and $\odot$ are defined as :

$(a, b) \oplus (c, d) = (a + c, b + d)$

and $(a, b) \odot (c, d) = (a \cdot c, b \cdot d)$,

then the system $(\mathbb{Z} \times 2\mathbb{Z}, \oplus, \odot)$ is :

(a) A ring with identity

(b) A ring with zero divisors

(c) An integral domain

(d) A field

31. The third column of the truth table

| P | Q | |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

is the truth functional rule for :

(a) $P \vee Q$

(b) $P \wedge Q$

(c) $P \Rightarrow Q$

(d) $P \vee Q \Rightarrow P \wedge Q$

32. Let P and Q be the mathematical statements. Then the negation of $P \Rightarrow Q$ is :

(a) $Q \Rightarrow P$

(b) $-P \Rightarrow Q$

(c) $P \Rightarrow -Q$

(d) $P \wedge -Q$

33. Let P and Q be the mathematical statements. Then the statement $(P \wedge -P) \Rightarrow Q$ is :

(a) A contradiction

(b) A tautology

(c) The negation of $P \Rightarrow Q$

(d) None of the above

34. Let P and Q be the mathematical statements. Which one of the following is a tautology ?

(a) $P \Rightarrow P \wedge Q$

(b) $P \Rightarrow P \vee Q$

(c) $P \Rightarrow -P$

(d) None of the above

35. Let P and Q be the mathematical statements. Then the contrapositive of $P \Rightarrow Q$ is :

(a) $-P \Rightarrow -Q$

(b) $-Q \Rightarrow -P$

(c) $P \Rightarrow -Q$

(d) $-P \Rightarrow Q$

36. The negation of "$\exists\ x \in \mathbb{Z}$ such that $\forall y \in \mathbb{Z}, x + y = 0$" is :

(a) $\exists\ x \in \mathbb{Z}$ such that $\forall\ y \in \mathbb{Z}$, $x + y \neq 0$

(b) $\exists\ x \in \mathbb{Z}$ such that $x + y \neq 0$ for some $y \in \mathbb{Z}$

(c) $\forall\ x \in \mathbb{Z}$, $\exists\ y \in \mathbb{Z}$ such that $x + y \neq 0$

(d) None of the above

37. For any three sets A, B and C following three statements are given :

(I) $A \cup B = A \cup C \Rightarrow B = C$

(II) $A \cap B = A \cap C \Rightarrow B = C$

(III) $A \cup B = A \cup C$ and $A \cap B = A \cap C \Rightarrow B = C$

Of these statements :

(a) I and II are correct but III is incorrect

(b) II and III are correct but I is incorrect

(c) I and III are correct but II is incorrect

(d) Only III is correct

38. If the numbers of elements in two disjoint sets are 2 and 3 respectively, then the number of elements in the union set of their power sets is :

(a) 12

(b) 11

(c) $2^5$

(d) 5

39. If there are 6 elements common between sets A and C and 4 elements common between sets B and D, then the number of elements common between the sets A × B and C × D is :

(a) 24

(b) 12

(c) 10

(d) 0

40. Which one of the following statements is incorrect ?

(a) $A - B = A \cap B^C$

(b) $A - B = A - (A \cap B)$

(c) $A - B = A - B^C$

(d) $A - B = (A \cup B) - B$

41. If sets A and B are defined as

$A = \{(x, y) : y = 1/x ; x(\neq 0), y \in \mathbb{R}\}$

$B = \{(x, y) : y = -x ; x, y \in \mathbb{R}\}$

then :

(a) $A \cap B = \phi$

(b) $A \cap B = A$

(c) $A \cap B = B$

(d) None of the above

42. Consider the following pairs of sets :

(I) $A \cup C ; B \cup D$

(II) $A \cup B ; C \cup D$

(III) $A \cup C ; B \cap D$

(IV) $A \cap C ; B \cap D$

where A, B, C and D are four sets such that $A \cap B = \phi = C \cap D$. Which of these pairs of sets are disjoint in general ?

(a) I and II

(b) II and III

(c) I and IV

(d) III and IV

43. If A and B are two square matrices such that $A \cdot B = A$ and $B \cdot A = B$, then :

(a) Both A and B are idempotent

(b) Only A is idempotent

(c) Only B is idempotent

(d) None of A and B is idempotent

44. Let det A = 5 where $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$, then

det $(2A)^{-1}$ is equal to :

(a) $\frac{1}{10}$

(b) $\frac{1}{25}$

(c) $\frac{1}{40}$

(d) $\frac{5}{2}$

45. If A is a real skew-symmetric matrix of order 5 × 5, then A is :

(a) Hermitian

(b) Orthogonal

(c) Non-singular

(d) Singular

46. If A is n × n matrix such that det A = 3 and det Adj A = 243, then n is equal to :

(a) 4

(b) 5

(c) 6

(d) 7

47. For two square matrices A and B of the same order consider the following assertions :

(I) Rank (A · B) = Rank A = Rank B

(II) Rank (A · B) = Rank A, if B is non-singular

(III) Rank (A · B) = Rank B, if B is non-singular

Which of these is / are correct ?

(a) Only I

(b) Only II

(c) I and II

(d) II and III

48. The system of linear equations $\lambda x + y + z = 1$, $x + \lambda y + z = \lambda$ and $x + y + \lambda z = \lambda^3$ does not have a solution if $\lambda$ is equal to :

(a) – 2

(b) – 1

(c) 0

(d) 1

49. In group theory, which one of the following statements is correct ?

(a) Abelian groups may have non-abelian subgroups.

(b) Non-abelian groups may have abelian subgroups.

(c) Cyclic groups may have non-cyclic subgroups.

(d) Non-cyclic groups can not have cyclic subgroups.

50. The number of subgroups of a cyclic group having 50 elements is :

(a) 6

(b) 5

(c) 10

(d) 2

51. Let A and B be the two independent events such that $Pr(A^c \cap B) = \frac{2}{15}$ and $Pr(A \cap B^c) = \frac{1}{6}$. Then Pr(B) is equal to :

(a) $\frac{1}{6}$

(b) $\frac{1}{5}$

(c) $\frac{3}{5}$

(d) $\frac{5}{6}$

52. A lot contains 20 articles. The probability that the lot contains exactly 2 defective articles is 0.4 and the probability that the lot contains exactly 3 defective articles is 0.6. Articles are drawn from the lot at random one by one without replacement and tested till all the defective articles are found. What is the probability that the testing procedure ends at the 12th testing ?

(a) $\frac{9}{19}$

(b) $\frac{99}{1900}$

(c) $\frac{10}{19}$

(d) $\frac{999}{1900}$

53. The order and degree of the differential equation of all tangent lines to the parabola $x^2 = 4y$ is :

(a) First order and second degree

(b) Second order and second degree

(c) Third order and first degree

(d) Fourth order and first degree

54. Solution of $\frac{dy}{dx} + 2xy = y$ is :

(a) $y = C e^{x - x^2}$

(b) $y = C e^{x^2} - x$

(c) $y = C e^{x}$

(d) $y = C e^{-x^2}$

55. The solution of the differential equation

$$\frac{dy}{dx} = \frac{y}{x} + \frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)}$$ is :

(a) $x\,\phi\left(\frac{y}{x}\right) = k$

(b) $\phi\left(y/_x\right) = kx$

(c) $y\,\phi\left(y/_x\right) = k$

(d) $\phi\left(y/_x\right) = ky$

56. The largest value of c such that there exists a differential function h(x) for $-c < x < c$ that is a solution of $\frac{dy}{dx} = 1 + y^2$ with $h(0) = 0$ is :

(a) $2\pi$

(b) $\pi$

(c) $\pi/_2$

(d) $\pi/_4$

57. A differential equation associated with the primitive $y = a + b\,e^{5x} + c\,e^{-7x}$ is :

(a) $y''' + 2y'' - y' = 0$

(b) $y''' + 2y'' - 35y' = 0$

(c) $4y''' + 5y'' - 20y' = 0$

(d) None of the above

58. The function $f(t) = \frac{d}{dt}\int_0^t \frac{dx}{1 - \cos t \cos x}$ satisfies the differential equation :

(a) $\frac{df}{dt} + 2f(t)\cot t = 0$

(b) $\frac{df}{dt} - 2f(t)\cot t = 0$

(c) $\frac{df}{dt} + 2f(t) = 0$

(d) $\frac{df}{dt} - 2f(t) = 0$

59. The radical axis of the circles, belongs to the coaxial system of circles whose limiting points are (1, 3) and (2, 6) is :

(a) $x - 3y + 15 = 0$

(b) $2x + 3y - 15 = 0$

(c) $x - 3y - 15 = 0$

(d) $4x + 3y - 15 = 0$

60. The product of the perpendiculars, drawn from any point on a hyperbola to its assymptotes is :

(a) $\frac{ab}{\sqrt{a} + \sqrt{b}}$

(b) $\frac{ab}{a^2 + b^2}$

(c) $\frac{a^2b^2}{a^2 + b^2}$

(d) $\frac{a^2 + b^2}{a^2b^2}$

61. The equation of the circle passing through (1, 0) and (0, 1) and having smallest possible radius is :

(a) $2x^2 + y^2 - 2x - y = 0$

(b) $x^2 + 2y^2 - x - 2y = 0$

(c) $x^2 + y^2 - x - y = 0$

(d) $x^2 + y^2 + x + y = 0$

62. If the latus rectum of an ellipse is one-half of its minor axis, then its eccentricity is :

(a) $\frac{1}{2}$

(b) $\frac{1}{\sqrt{2}}$

(c) $\frac{\sqrt{3}}{2}$

(d) $\frac{\sqrt{3}}{4}$

63. The equation of the lines through the origin and perpendicular to the lines $ax^2 + 2hxy + by^2 = 0$ are given by :

(a) $ax^2 - 2hxy + by^2 = 0$

(b) $bx^2 - 2hxy + by^2 = 0$

(c) $hx^2 - 2bxy + ay^2 = 0$

(d) $bx^2 - 2hxy + ay^2 = 0$

64. Focus of the parabola $(y - 2)^2 = 20 (x + 3)$ is :

(a) (– 3, 2)

(b) (3, – 2)

(c) (2, – 3)

(d) (2, 2)

65. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ then :

(a) $\vec{a}$ is perpendicular to $\vec{b}$

(b) $\vec{a} = \frac{1}{2}\vec{b}$

(c) Angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$

(d) $\vec{a}$ is parallel to $\vec{b}$

66. The vectors $6\vec{i} - 2\vec{j} + 3\vec{k}$, $2\vec{i} + 3\vec{j} - 6\vec{k}$ and $3\vec{i} + 6\vec{j} - 2\vec{k}$ from a triangle which is :

(a) Isosceles

(b) Right angled

(c) Obtuse angled

(d) Equilateral

67. The condition that $ax^2 + by^2 = 1$ and $Ax^2 + By^2 = 1$ should cut orthogonally is :

(a) $\frac{1}{a} + \frac{1}{A} = \frac{1}{b} + \frac{1}{B}$

(b) $\frac{1}{a} - \frac{1}{A} = \frac{1}{b} - \frac{1}{B}$

(c) $\frac{1}{a} + \frac{1}{b} = \frac{1}{A} + \frac{1}{B}$

(d) $\frac{1}{A}+\frac{1}{B}=a+b$

68. We know $f'(c)=\frac{f(b)-f(a)}{b-a}$, $a<c<b$.

Here the existence of 'c' is guaranteed by 'Mean value theorem'. But this c is not unique and can be examined by the function sin (x) for :

(a) $x\in[-\pi, \pi]$

(b) $x\in[\pi, 2\pi]$

(c) $x\in[0, \pi]$

(d) $x\in\left[\frac{\pi}{4},\frac{\pi}{2}\right]$

69. If $u=\frac{xy}{x+y}$, then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ is equal to :

(a) 1

(b) 0

(c) $-u$

(d) u

70. Given f(x, y) =

$$\begin{cases}\frac{x^3}{x^2+y^2} & \text{if } (x, y)\neq(0, 0)\\ 0 & \text{if } (x, y)=(0, 0)\end{cases}$$ then :

(a) $f_x(0, 0)=0$

(b) $f_x(0, 0)=1$

(c) $f_x(0, 0)=2$

(d) $f_x(0, 0)$ does not exist

71. If f(0) = 5 and f(x) < 5 for all $x\neq 0$ then which of the following is true ?

(a) $f'(0)=0$

(b) $f'(0)=5$

(c) $f'(0)=-5$

(d) $f'(0)=1$

72. For the function $f(x)=\sqrt{x}-x$, $0\leq x\leq 1$, which of the following is true ?

(a) Rolle's theorem is not applicable to f and yet $\exists\, c\in(0, 1)$ such that $f'(c)=0$

(b) Rolle's theorem is not applicable to f since f is not differentiable at x = 0

(c) Rolle's theorem is not applicable to f and hence there is a $c\in(0, 1)$ such that $f'(c)=0$

(d) None of the above

73. The value of the integral $\int_{-a}^{a}\sqrt{1-\frac{x^2}{a^2}}\,dx$ is :

(a) $\pi a^2$

(b) $\pi a$

(c) $2\pi a$

(d) $\frac{\pi}{2}a$

74. Given $\int_0^{\sqrt{3}} \frac{dx}{1+x^2} = 2\int_a^{\sqrt{3}} \frac{dx}{1+x^2}$, then the value of 'a' is :

(a) $\frac{1}{2}$

(b) $\sqrt{3}$

(c) $\frac{1}{\sqrt{3}}$

(d) $\frac{\pi}{3}$

75. The area of the smaller region lying above the x-axix and included between the circle $x^2 + y^2 = 2x$ and the parabola $y^2 = x$ is :

(a) $\frac{2}{3} - \frac{\pi}{4}$

(b) $\frac{\pi}{4} - \frac{2}{3}$

(c) $\frac{\pi}{4} + \frac{2}{3}$

(d) $\frac{\pi}{4}$

76. Which of the following is the area of a cardioid $r = a(1 - \cos\theta)$ ?

(a) $\frac{3}{4}a^2\pi$

(b) $\frac{1}{2}a^2\pi$

(c) $\frac{2}{3}a^2\pi$

(d) $\frac{3}{2}a^2\pi$

77. If $I = \int_0^2 \frac{x+7}{6+x-x^2}dx$ then I is equal to :

(a) $-\log 4$

(b) 2

(c) $-2$

(d) $\log 4$

78. What is the area between $y = \sin x$ and $y = \cos x$ in square units, where $x \in [0, 2\pi]$ ?

(a) 3

(b) 0

(c) $4\sqrt{2}$

(d) $2\sqrt{2}$

79. Ten guests are to be seated in a row of which three are women. The women insist on sitting together while two of the men refuse to take consecutive seats. In how many ways can the guests be seated ?

(a) 181440 ways

(b) 181430 ways

(c) 181420 ways

(d) 181410 ways

80. There are n coplanar straight lines, no. two being parallel and no. three are concurrent. How many different new straight lines will be formed by joining the intersection points of the given lines ?

(a) $\frac{1}{4}n(n-1)(n-2)(n-3)$

(b) $\frac{1}{6}n(n-1)(n-2)(n-3)$

(c) $\frac{1}{8}n(n-1)(n-2)(n-3)$

(d) $\frac{1}{10}n(n-1)(n-2)(n-3)$

81. If ${}^{(k+5)}P_{(k+1)} = \frac{11(k-1)}{2} \times {}^{(k+3)}P_{(k)}$, then the value of k is equal to :

(a) 5, 6

(b) 6, 7

(c) 7, 8

(d) 8, 9

82. If $n \in N$, $c_k = {}^nC_k$, then the value of $\sum_{k=1}^{n}\left(\frac{c_k}{c_{k-1}}\right)^2$ is :

(a) $\frac{1}{12}n(n+1)(n+2)$

(b) $\frac{1}{12}n(n+1)^2(n+2)$

(c) $\frac{1}{12}n(n+1)^2(n+2)^2$

(d) $\frac{1}{12}n^2(n+1)^2(n+2)^2$

83. The resultant of two equal forces, acting at a point at an angle $\alpha$, is in the direction :

(a) Of one force

(b) Perpendicular to one force

(c) Along $\alpha/2$

(d) Along $\pi - \alpha$

84. The necessary condition that three forces not concurrent, acting on a rigid body, are in equilibrium is :

(a) The forces are parallel

(b) Two are parallel and third is opposite

(c) The forces are at right angles

(d) The forces are coplanar

85. In case of friction, the coefficient of friction is equal to :

(a) Reaction of the body

(b) Inverse of the angle of friction

(c) Angle of friction

(d) Tangent of angle of friction

86. A couple is formed by :

(a) Two equal forces acting in any direction

(b) Two unequal forces (unlike and parallel)

(c) Two equal unlike parallel forces

(d) Two equal like parallel forces

87. A number '64B' in Hexadecimal representation is written in Octal representation as :

(a) 3123

(b) 1231

(c) 3131

(d) 3113

88. Binary representation of a decimal number 11.625 is :

(a) 1011.11

(b) 1011.011

(c) 1011.101

(d) 1101.011

89. Convergence rate of Newton-Raphson method in finding the root of a non-linear equation is :

(a) 2.0

(b) 1.5

(c) 2.5

(d) 1.0

90. Runge-Kutta methods are used to solve :

(a) Integrals

(b) Ordinary differential equations

(c) Determinants

(d) Partial differential equations

91. The term 'quadrature' relates to :

(a) Roots of quadratic equation

(b) Area under a curve

(c) Length of a curve

(d) Quadrants in a plane

92. The finite-difference operator '$\nabla$' means :

(a) $\nabla f(x_i) = f(x_{i+1}) - f(x_i)$

(b) $\nabla f(x_i) = f(x_i) - f(x_{i-1})$

(c) $\nabla f(x_i) = f(x_{i+1/2}) - f(x_{i-1/2})$

(d) $\nabla f(x_i) = [f(x_{i+1}) - f(x_{i-1})] / 2$

93. If $\vec{a}$ and $\vec{b}$ are unit vectors and $\theta$ is the angle between them, then $|\vec{a} + \vec{b}|$ is equal to :

(a) $2\sin\frac{\theta}{2}$

(b) 2 units

(c) $2 \cos \theta$

(d) $2 \cos \frac{\theta}{2}$

94. The direction cosines $\ell$, m, n of two lines are connected by the relations $\ell + m + n = 0$ and $2\ell m + 2\ell n - mn = 0$. Then they are :

(a) $\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}} ; \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}$

(b) $\frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{-1}{\sqrt{6}} ; \frac{2}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}$

(c) $\frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{-1}{\sqrt{6}} ; \frac{2}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}$

(d) None of the above

95. The angle between the pair of planes $2x - y + 2z = 3$ ; $3x + 6y + 2z = 4$ is :

(a) $\cos^{-1}\left(\frac{4}{21}\right)$

(b) $\tan^{-1}\left(\frac{2}{21}\right)$

(c) $\cos^{-1}\left(\frac{2}{21}\right)$

(d) None of the above

96. The equation of the sphere through the four points (4, – 1, 2), (0, – 2, 3), (1, – 5, – 1) and (2, 0, 1) is :

(a) $x^2 + y^2 + z^2 + 4x - 6y + 2z - 5 = 0$

(b) $x^2 + y^2 + z^2 - 4x + 6y - 2z + 5 = 0$

(c) $x^2 + y^2 + z^2 - 4x - 6y - 2z - 5 = 0$

(d) None of the above

97. With a given velocity of projection, with angle of projection $\alpha$, the horizontal range is same for two angles :

(a) $\alpha, \frac{\pi}{2} - \alpha$

(b) $\alpha, \frac{\pi}{2} + \alpha$

(c) $\alpha, \pi - \alpha$

(d) $\alpha, \pi + \alpha$

98. If the radial and transverse velocity of a particle are proportional then the path of the partcle is :

(a) Straight line

(b) Equiangular spiral

(c) Circle

(d) Parabola

99. Moment of inertia of a rod of length 2a about a perpendicular line through one end is :

(a) $\frac{Ma^2}{3}$

(b) $\frac{Ma^2}{4}$

(c) $\frac{4Ma^2}{3}$

(d) $\frac{Ma^2}{9}$

where M is the mass of rod.

100. In simple Harmonic Motion, the acceleration is propotional to :

(a) Distance from a fixed point

(b) Inverse distance from a fixed point

(c) A constant

(d) None of the above

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## SPACE FOR ROUGH WORK